Comparison of Calculations of Neutron Cross Sections on Deformed Nuclei Between Hauser-Feshbach and Deformed Hauser-Feshbach Models

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Conventional Hauser-Feshbach

$\square ab = \square \square 2/(2I1+1)(2I2+1) \ \sum J \square \uparrow (2J+1) Tin\, Tout\, \square (U\uparrow', J\uparrow', \square \uparrow') / \sum \uparrow Tout\, \square (u\uparrow'', J\uparrow'', \square \uparrow'')$

- Have sums over the final angular momentum values.
- Integrals over outgoing energies.
- No allowance for dependence on K
- Formalism enforces spherical symmetry even on deformed nuclei.

Analogous problem in 1960's

- Studies of $^{59}Co(p,p')$, $^{59}Co(p,\alpha)$, $^{56}Fe(\alpha,p)$, $^{56}Fe(\alpha,\alpha')$ at same center of mass energy
- $R = \square(\square,\square') \square(p,p') / \square(\square,p) \square(p,\alpha)$
- Should be 1 by the independence hypothesis
- Hauser-Feshbach results in R= 1.15 change in the p and α branching ratio with *J*.
- The α and proton bombardments give different *J* distributions.
- Experiment gave R= 1.4
- ^{59}Co + p gives isospin 2 and 3
- ^{56}Fe + α gives only isospin 2
- The branching ratio depends on isospin as well as *J*.

A New HF Formulation

- Add sum over T_I (Compound Nucleus).
- Make final level densities a function of final T_I
- Sum over Final T_I
- Replace Transmission Coefficients with
- $\tau = T < T_{I1}\ T_{I1Z}\ T_{I2}\ T_{I2Z} \mid T_{I3}\ T_{I3Z} >^2$
- Result: R=1.65
- Can reproduce experimental R= 1.4 by invoking isospin mixing

- Density of T_I levels is much larger than T_{I+1} mixing is one way.
- $0 \leq \mu \leq 1$ $\mu = 1$ recovers the no isospin result.

Analogous Situation for K

1. Add sum over compound K

2. Add sum over final K'.

3. Enter level density as a function of K'.

4. T is replaced by :

$\tau = T \langle J_1 K_1 J_2 K_2 | J_3 K_3 \rangle^2$

Results can be understood to be consequences of:

1. K selection rules

- For example in decay from 3^- to 2^+ state
  - In spherical basis, E1 decay can occur
  - In Deformed Basis $|\Delta K| \leq 1$ for E1
    - $J = 3^-$ $K = 3$ level needs E3 to go to $J = 2^+$ $K = 0$
    - $J = 3^-$ $K = 3$ level could decay to $J = 2^+$ $K = 2$ level with E1

- $J = 5^{+}/2$ level can decay with S wave neutron to 2^{+} in spherical basis.

- $J = 5^{+}/2$ level cannot decay with S wave neutron to J = 2, K = 0 in deformed basis unless K = 1/2.

2. Level of spin J in spherical basis is 2J+1 degenerate, level of spin J is singly degenerate for K = 0, and doubly degenerate if $K \neq 0$ in a deformed basis.

   - in spherical basis a 4^{+} level has 9 states or ~twice the degeneracy of a 2^{+} level with 5 states (9/5).

   - In deformed basis: the ratio of states between a 4^{+} and 2^{+} can be 1 if the K values are the same.

- Ratio can be 2 if for 4^+, $K \neq 0$ and 2^+, $K = 0$
- Ratio can be 0.5 if for 4^+, $K = 0$ and 2^+, $K \neq 0$
- Cross sections for nearby levels of same J^π can differ by a factor of 2 if one has $K = 0$ and the other has $K \neq 0$.
- For odd A, $K = 0$ is not allowed. All degeneracies are equal!

Calculations have been done for:

- n+ ^{168}Er
- n + ^{183}W
- n + ^{182}W
- n + ^{25}Mg
- α + ^{22}Ne
- For:
  - $0.5 \text{ MeV} \leq E\downarrow n \leq$ 15 MeV and
  - $3\ MeV \leq E\downarrow\alpha \ \leq 15\ MeV$

TABLE III. Cross-section ratio values for $n + {}^{182}W$.

Reaction	Bombarding energy (MeV)					
	0.5	1	4	7	10	14
(n, α)	1.67	1.77	1.85	1.7	1.51	1.3
(n, p)			0.98	1.0	1.0	1.0
$(n, 2n)$						0.97
$(n\alpha, \alpha n)$						1.52
(n, γ)	1.12	1.07	1.03	1.01		
The following are the ten lowest levels of ^{182}W:						
$(n, n')\ J = 0\ K = 0^+$	0.56	0.52	0.36	0.30	0.27	0.24
$(n, n')\ J = 2\ K = 0^+$	2.12	2.21	1.25	1.42	1.26	1.14
$(n, n')\ J = 4\ K = 0^+$	3.25	3.44	2.84	2.3	2.15	1.96
$(n, n')\ J = 6\ K = 0^+$		3.56	3.75	3.16	2.93	2.71
$(n, n')\ J = 0\ K = 0^+$			0.377	0.322	0.273	0.24
$(n, n')\ J = 8\ K = 0^+$			3.6	3.87	3.59	3.58
$(n, n')\ J = 2\ K = 0^+$			1.68	1.45	1.28	1.14
$(n, n')\ J = 2\ K = 2^+$			0.75	0.67	0.59	0.54
$(n, n')\ J = 1\ K = 1^-$			0.54	0.46	0.40	0.35
$(n, n')\ J = 3\ K = 2^+$			0.99	0.89	0.8	0.73

General Trends

- Continuum effects small on shape and magnitude.
- Average J of continuum states changes by 20-30%
- Large J resolved states reduced.
- Small J states enhanced.

Additional Discovery

- Bethe introduced [illegible]$=\langle J_z^2\rangle^{1/2}$
- Bohr and Mottelson give:

$$[\text{illegible}]_{def}(u) = \sigma_{\perp}^{2}\rho_{spherical}(u)$$

- This is often used to determine $\rho(u)$ at the binding energy from the $½^{+}$ density.
- Calculations show this is not correct.
- Adding rotational bands skews the J distribution towards large J.
- Correct formula is not as simple and form is given in S.M. Grimes, Phys. Rev. C**88**, 024613 (2013).
- The correct form can change the ratio $\rho(u,1/2)/\rho(u,total)$ by 30-35%.

- Comparison of results with no K mixing, moderate K mixing and complete K mixing shows low sensitivity to mixing.

- Complete K mixing does **not** restore results to traditional Hauser-Feshbach .

- No calculations have been made yet for fissionable nuclei.